ABSTRACTS

V. Maiorov (Technion, Israel)

Harmonic analysis on the ball and approximation by ridge-functions

---

A ridge function is any multivariate real-valued function $F : \mathbb{R}^d \to \mathbb{R}$ of the form

$$F(x_1, ..., x_d) = f(a_1x_1 + \cdots + a_dx_d) = f(a \cdot x).$$

The vector $a = (a_1, ..., a_n)$ is called the direction. In our report we consider the sets $\mathcal{R}_n$ generating by linear combinations of all possible n ridge-functions. We review the results obtained on last time and connecting with approximation of different functions (smoothness, harmonic, convex and other) by the sets $\mathcal{R}_n$.

The sets $\mathcal{R}_n$ with variable and fixed directions a are considered. Exact asymptotical estimations for best approximation of the function classes are formulated. The cases of efficiency of approximation by ridges with variable directions as compared with fixed are considered. The main tool of investigation of these problems is harmonic analysis connecting with construction of the special orthogonal basis of polynomials on the unit ball.

* * *

E.Kopecka (University of Innsbruck, Austria)

Convergence of products of orthogonal projections

e-mail: Eva.Kopecka@uibk.ac.at

We will investigate when a sequence of orthoprojections of a point in a Hilbert space on a finite family of closed subspaces, or more generally, closed convex subsets, converges. We will explain non-trivial examples when the sequence does not converge.

* * *

A.A. Schkalikov (Lomonosov Moscow State University)

Analytic problems of similarity of PT-symmetric self-adjoint Hamiltonians

e-mail: shkalikov@mi.ras.ru; ashkaliko@yandex.ru

Our report is about so-called PT-symmetric operators. Such operators are characterized by spatial variable symmetry and conjugation symmetry. Operators like

$$T = -\frac{d^2}{dx^2} + \varepsilon P(x), \quad \varepsilon > 0,$$

are the classical example of PT-symmetric operators in the space $L_2(-a, a)$, $0 < a \leqslant \infty$, with the condition $P(x) = -\overline{P(-x)}$. Spectrum of that operators has real-axis symmetry, but,

indeed, it is not always real. In the case of finite interval the spectrum is always discrete (if P is not a singular potential), moreover, we will show, that for sufficiently small ε the spectrum is real, and PT-symmetric operator T is similar to self-adjont one. If the ε grows, then complex eigenvalues are appeared and their number is increasing according to growth of ε.

Following problems are arising: how change the characteristics of operator T on the assumption of ε? Is it possible to estimate the critical values of ε, at which complex eigenvalues appear? We will give answers for these questions during the investigation of one model problem.

Report is based on joint work with S.N. Tymanov.

* * *

Yu.V.Nesterenko (Lomonosov Moscow State University)

Estimations of linear forms at values of analytic functions

Ю.В.Нестеренко (Мех-Мат, МГУ)

Оценки линейных форм от значений аналитических функций

e-mail: nester@mi.ras.ru

Доклад будет посвящен результатам Е.М. Никишина про линейную независимость значений полилогарифмов в рациональных точках и о последовавших затем продвижениях в этой области. В частности, будет рассказано о том, как доказывается существование бесконечного количества иррациональных среди значений дзета-функции Римана в нечётных положительных точках, оценки снизу для линейных форм с целыми коэффициентами от значений некоторых гипергеометрических функций и другие результаты.

* * *

M. Yattselev (IUPUI, Indiana, USA)

Hermite-Pade approximants for a pair of Cauchy transforms with overlapping symmetric supports

e-mail: maxyatts@iupui.edu

Hermite-Padé approximants of type II are vectors of rational functions with common denominator that interpolate a given vector of power series at infinity with maximal order. We are interested in the situation when the approximated vector is given by a pair of Cauchy transforms of smooth complex measures supported on the real line. The convergence properties of the approximants are rather well understood when the supports consist of two disjoint intervals (Angelesco systems) or two intervals that coincide under the condition that the ratio of the measures is a restriction of the Cauchy transform of a third measure (Nikishin systems). In this talk I will describe the case where the supports form two overlapping intervals (in a symmetric way) and the ratio of the measures extends to a holomorphic function in a region that depends

on the size of the overlap. Hermite-Padé approximants for such functions exhibit both the presence of the divergence domains (the divergence domains appear for Angelesco systems but are not present for Nikishin systems), and the presence of overinterpolation (a feature peculiar for Nikishin systems but not for Angelesco systems).

Joint work with Alexander I. Aptekarev and Walter Van Assche.

* * *

B. Beckermann (Lille University, France)

Vector equilibrium problems for Nikishin systems with touching or overlapping plates

bbecker@math.univ-lille1.fr

Given two closed Δ_1, Δ_2, two functions $Q_j : \Delta_j \mapsto \mathbb{R} \cup \{+\infty\}$ and parameters $\rho_j > 0$, $j = 1, 2$, the Nikishin vector equilibrium problem looks for a vector of measures minimizing the vector energy

$$J(\mu_1, \mu_2) = \sum_{j,k=1}^{2} C_{j,k} \int \log \frac{1}{|x-y|} d\mu_j(x) d\mu_k(y) + 2 \sum_{j=1}^{2} \int Q_j(x) d\mu_j(x)$$

over all measures μ_j supported on Δ_j with prescribed masses $\|\mu_j\| = \rho_j$, with the Nikishin interaction matrix

$$C = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

The classical analysis going back to Nikishin supposes that the two Δ_j are disjoint, there are however several examples in the literature where the Δ_j are touching intervals [5]. We will report about such an application going back to Eckhoff, Driscoll and Fornberg, namely the reconstruction of a real-valued function with one known discontinuity from it's Fourier coefficients and its relation with Hermite-Padé approximants for Nikishin systems [2, 3]. Subsequently we show that the above extremal problem has a unique solution even if the plates Δ_j are touching or overlapping [1, 4].

Joint work with Ana Matos (Lille, Franck Wielonsky (Marseille) and Valery Kalyagine (Nizhny Novgorod).

References

[1] B. Beckermann, V. Kalyagine, A.C. Matos, F. Wielonsky: Equilibrium problems for vector potentials with semidefinite interaction matrices and constrained masses, Constr. Approx.37 (2013) 101-134.

[2] B. Beckermann, V. Kalyagine, A.C. Matos, F. Wielonsky: How well does the Hermite-Padé approximation smooth the Gibbs phenomenon ? Math. Comp. 80 (2011), 931-958.

[3] B. Beckermann, A.C. Matos, F. Wielonsky: Reduction of the Gibbs phenomenon for smooth functions with jumps by the ϵ-algorithm, J. Comput. Appl. Math. 219 (2008), 329-349.

[4] A. Hardy, A. Kuijlaars, Weakly Admissible Vector Equilibrium Problems, Journal of Approximation Theory 164 (2012), 854-868

[5] E. M. Nikishin, V. N. Sorokin, Rational Approximations and Orthogonality, Transl. Amer. Math. Soc., Vol. 92, Providence, R.I. (1991).

* * *